

Mensuration

A comprehensive guide to key concepts in geometry – covering Pythagoras' Theorem, angle properties, triangles, circles, polygons, similar solids, and set theory.

MATHEMATICS

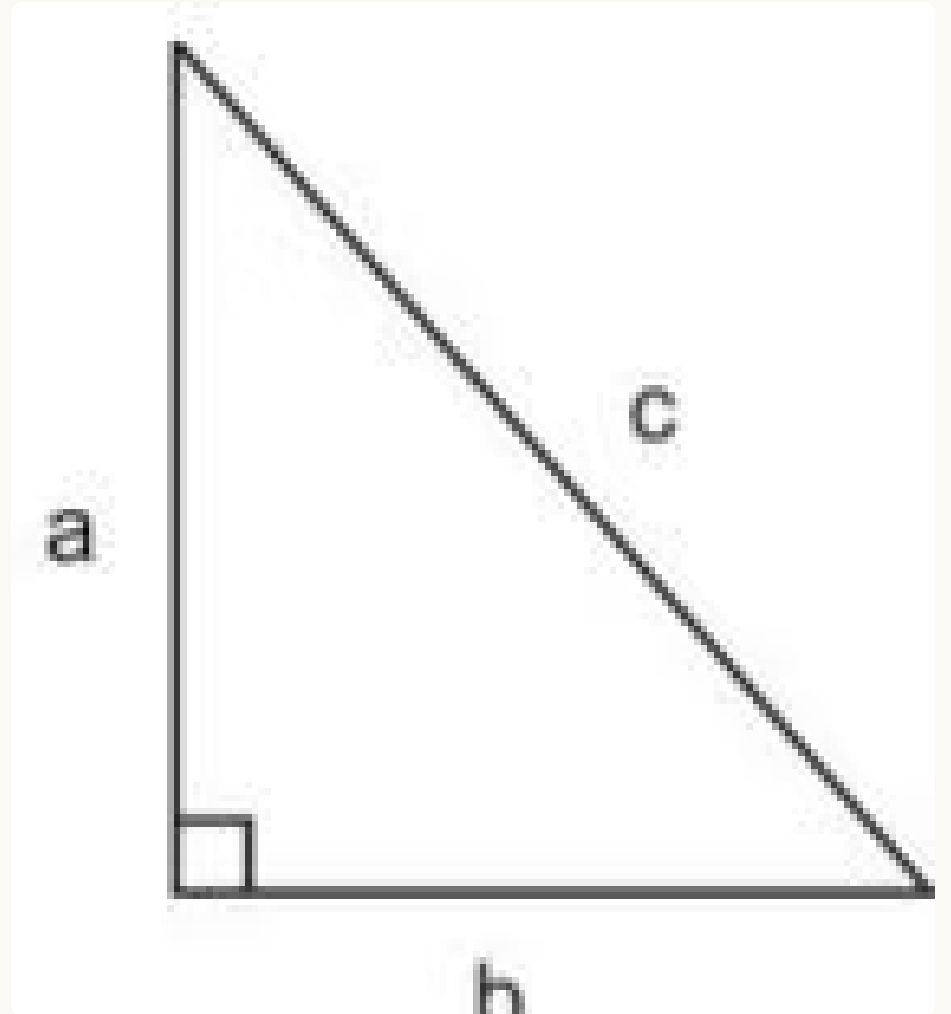
GEOMETRY & SETS

Pythagoras' Theorem

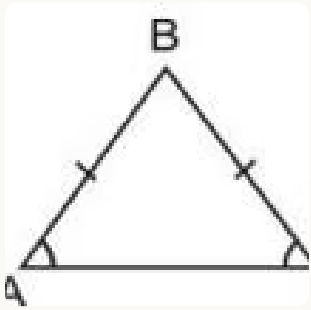
For all **right-angled triangles**, "the square on the hypotenuse is equal to the sum of the squares on the other two sides."

$$c^2 = a^2 + b^2$$

Where **c** is the hypotenuse (the longest side, opposite the right angle), and **a** and **b** are the two shorter legs.

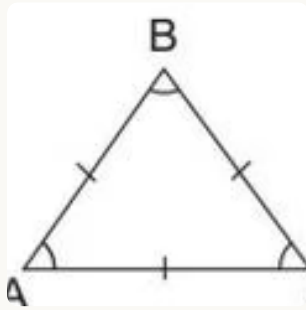


Types of Triangles



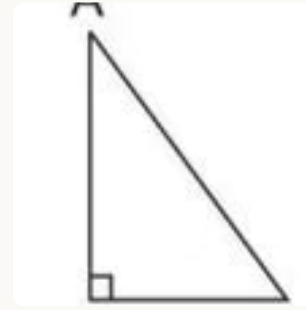
Isosceles

2 equal sides and 2 equal angles. $AB = AC$ and $\angle ABC = \angle BCA$



Equilateral

3 equal sides and 3 equal angles.
 $AB = BC = CA$ and $\angle ABC = \angle BCA = \angle CAB$



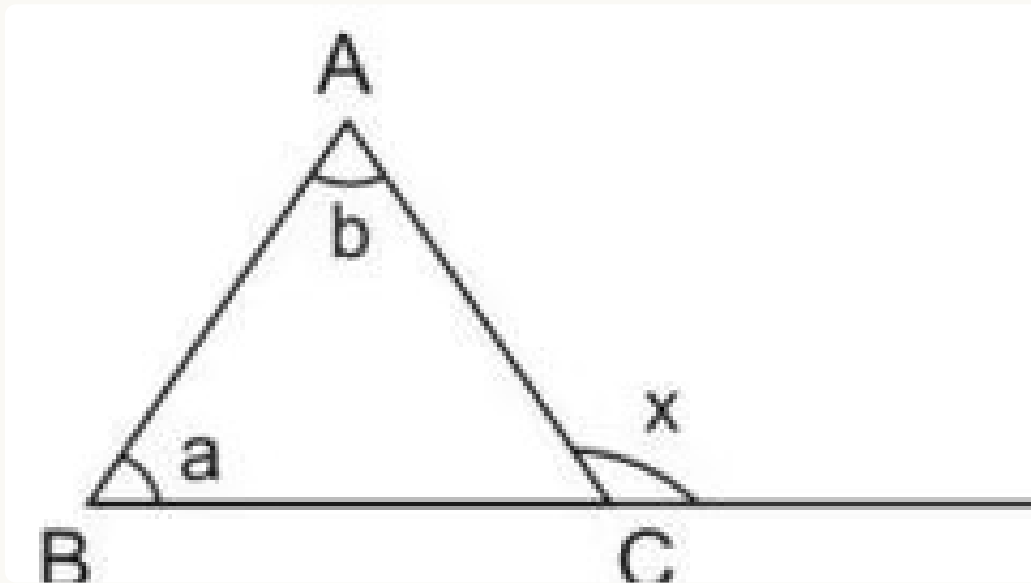
Right-Angled

One interior angle equals 90° . $\angle ABC = 90^\circ$

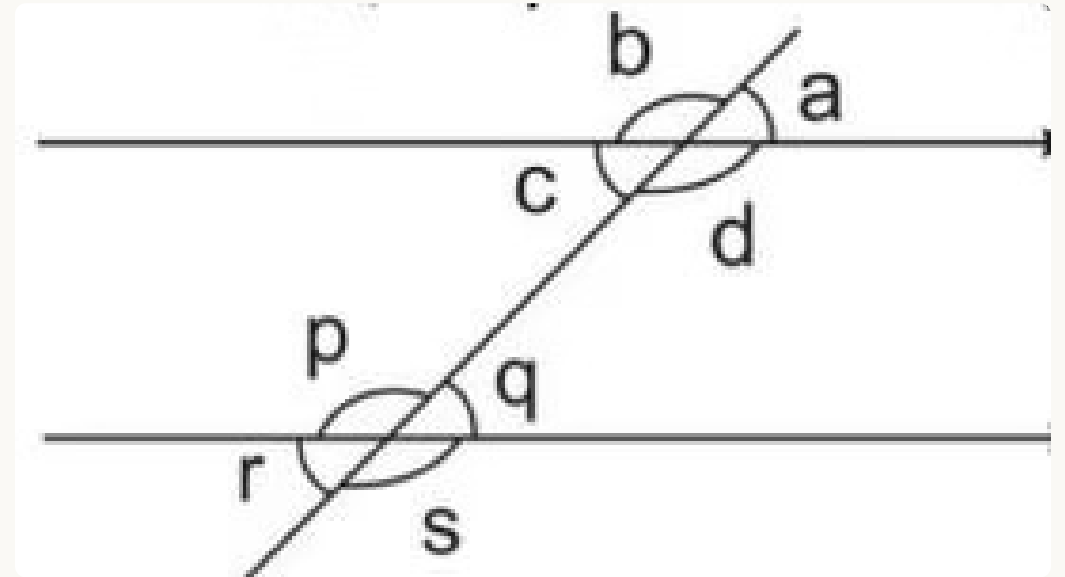
Angle Properties of Triangles & Parallel Lines

Triangle Angle Properties

- Sum of angles in a triangle = 180°
- Greatest angle is opposite the longest side
- Exterior angle = sum of the two opposite interior angles: $x = a + b$



Parallel Lines & Transversal

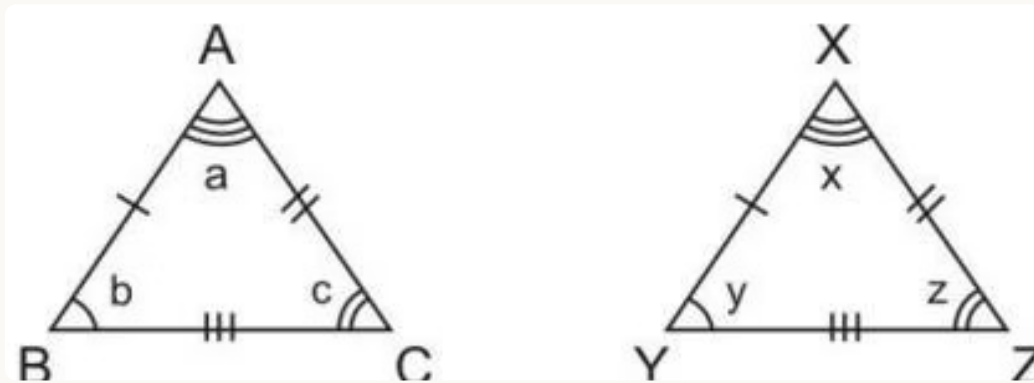


- **Vertically opposite:** $a = c$; $b = d$; $p = r$; $q = s$
- **Corresponding:** $a = q$; $b = p$; $c = r$; $d = s$
- **Alternate:** $c = q$ and $d = p$

Congruent & Similar Triangles

Congruent Triangles

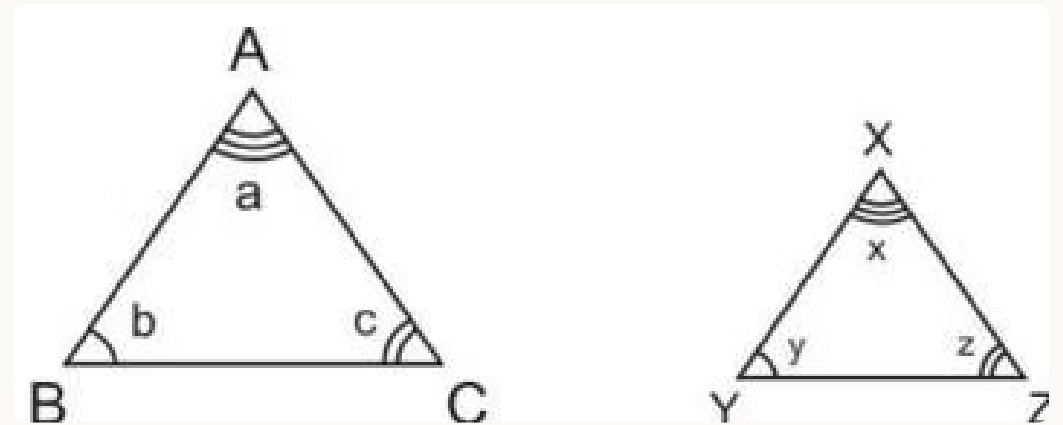
Two triangles are congruent if they are equal in every aspect (sides and angles).



Similar Triangles

Two triangles are similar if corresponding angles are equal and corresponding sides are proportional.

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$



Conditions: (i) all corresponding angles equal, (ii) all ratios of corresponding sides equal, or (iii) two sides proportional with included angle equal.

① Areas of similar triangles: $\frac{\text{Area}_1}{\text{Area}_2} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

Polygons, Types of Angles & Similar Solids

Polygons

- Exterior angles sum = 360°
- Interior angles sum = $(n - 2) \times 180^\circ$
- Regular polygon: each exterior angle = $\frac{360}{n}$
- Triangle (3), Quadrilateral (4), Pentagon (5), Hexagon (6), Heptagon (7), Octagon (8), Nonagon (9), Decagon (10)

Types of Angles

- **Acute:** $\theta < 90^\circ$
- **Obtuse:** $90^\circ < \theta < 180^\circ$
- **Reflex:** $180^\circ < \theta < 360^\circ$

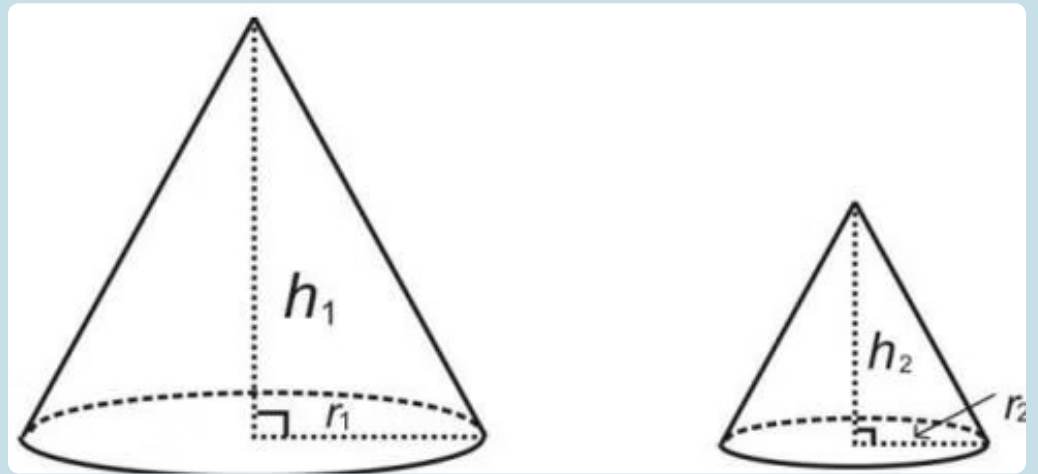
Similar Solids

If two objects are similar with ratio of corresponding sides k :

- Ratio of areas = k^2
- Ratio of volumes = k^3

$$\frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{h_1}{h_2}$$

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \quad \frac{V_1}{V_2} = \frac{r_1^3}{r_2^3}$$



Circle Theorems

Central & Inscribed Angles

The angle subtended by an arc at the centre is **twice** the angle subtended at the circumference.

Same Segment

Angles subtended by an arc in the **same segment** of a circle are equal.

Angle in a Semicircle

The angle in a semicircle is a right angle. $\angle APB = 90^\circ$

Cyclic Quadrilateral

Opposite angles of a cyclic quadrilateral are supplementary:

$$A + C = 180^\circ \text{ and } B + D = 180^\circ$$